Heat Transfer - Concepts

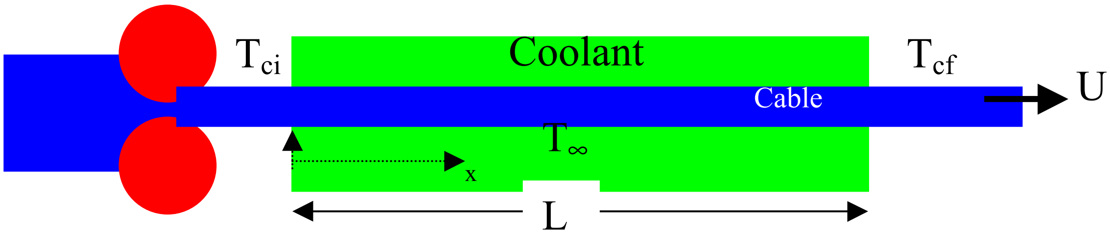


Figure 1: Schematic of the cable cooling process.

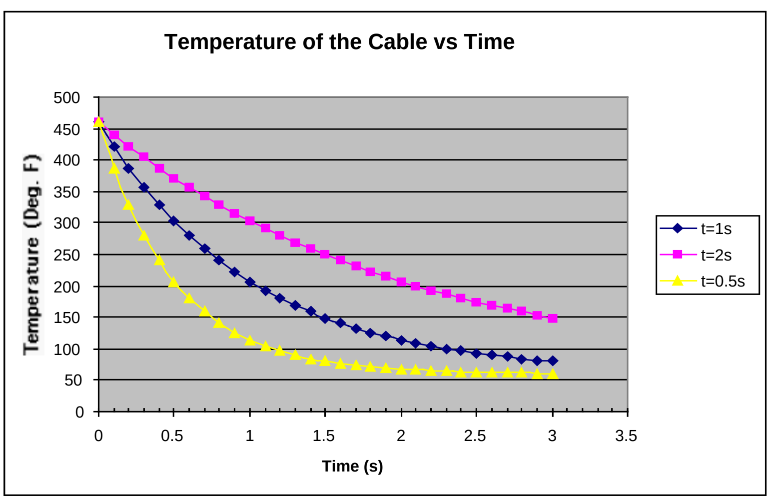
Overview

Using Eqs. 8 & 9, we can predict the cooling characteristics of the polyethylene jacket.

If we rearrange Eq. (8) as

$$T_{cf} = e^{-t/\tau}(T_{ci} - T_{\infty}) + T_{\infty} \qquad \text{Eq. (10)}$$

For the following temperature conditions T_{ci}=460°F, T_{∞}=60°F, we can plot the T_{cf} as a function of time. Plot shows cooling curves for τ=0.5s, τ=1s, τ=2s. From the plot we can determine that the cable will cool to 100°F in 2.3s if the τ=1s. If the time constant is large the cable will cool slowly, and if the time constant is small the cable will cool faster.



Assumptions:

- We will assume most of the heat loss take place due quenching in the coolant. Therefore, the heat loss is primarily due to convective heat transfer between the polyethylene jacket and coolant.
- The cable consists of a heated polyethylene jacket with a non-conductive core (fiber cables). Therefore, we will neglect radial heat conduction from the outer polyethylene jacket to the inner core.
- We will assume the thickness of the polyethylene jacket is small. Therefore, we will assume the surface temperature of the polyethylene jacket is equal to the radial mean temperature at any location along the cable (T_c).
- We will also neglect conduction along the length of the cable through the cross-sectional area of the polyethylene jacket, since the cross-sectional area is very small and the temperature gradient is also small.

Symbols

q - convective heat (W)
h - Local heat transfer coefficient varies with x ($W/m^2.K$)
h_{avg} - Average heat transfer coefficient ($W/m^2.K$)
T_c - Local cable temperature varies with x (K)
T_∞ - Mean Coolant Temperature – constant (K)
$\dot{m}$ - Mass flow rate of polyethylene jacket (kg/s)
A - Cross-sectional area of the polyethylene jacket (m^2)
ρ_p - Density of the polyethylene jacket (kg/m^3)
c - specific heat capacity of the polyethylene jacket (J/kg.K)
P - Perimeter of the cable - πD (m)
D - Diameter of the cable (m)
L - Length of the coolant trough (m)
t - Time (s)
U - Cable velocity – L/t (m/s)
τ - Time constant (s)

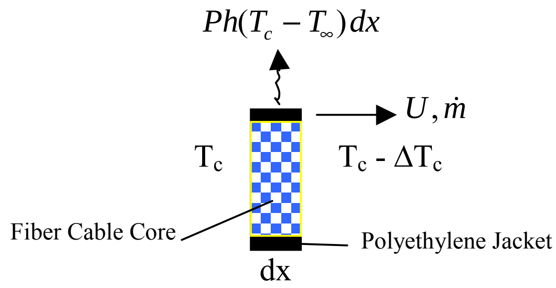


Figure 2: Control volume for the cable showing heat transfer to the coolant

Applying Newton's Law of cooling to the control volume, we can express the temperature drop along the cable as being equal to the heat loss to the coolant.

$$-\dot{m}c\,dT_c = Ph(T_c - T_\infty)dx$$
$$-\frac{dT_c}{dx} = \frac{Ph}{\dot{m}c}(T_c - T_\infty) \quad \text{Eq. (1)}$$

Defining $\Delta T = T_c - T_\infty$ Eq. (2)

We can express Eq. (1) as

$$-\frac{d\Delta T}{dx} = \frac{ph}{\dot{m}c}\Delta T \quad \text{Eq. (3)}$$

Integrating Eq. (3) for the initial and final conditions

$$\int_{\Delta T_i}^{\Delta T_f} \frac{d\Delta T}{\Delta T} = -\frac{P}{\dot{m}c}\int_{x=0}^{x=L} h\,dx \quad \text{Eq. (4)}$$

In Eq. (4) $\Delta T_f = T_{cf} - T_\infty$ and $\Delta T_i = T_{ci} - T_\infty$ Eq. (5)

$$\frac{\Delta T_f}{\Delta T_i} = e^{-\frac{PLh_{avg}}{\dot{m}c}} \quad \text{Eq. (6)}$$

Defining $\dot{m} = A\rho_p \frac{L}{t}$ Eq. (7)

By substitute Eq. (7) and Eq. (5) into Eq. (6), we can express Eq. (6) as

$$\frac{T_{cf} - T_\infty}{T_{ci} - T_\infty} = e^{-\frac{t}{\tau}} \quad \text{Eq. (8)}$$

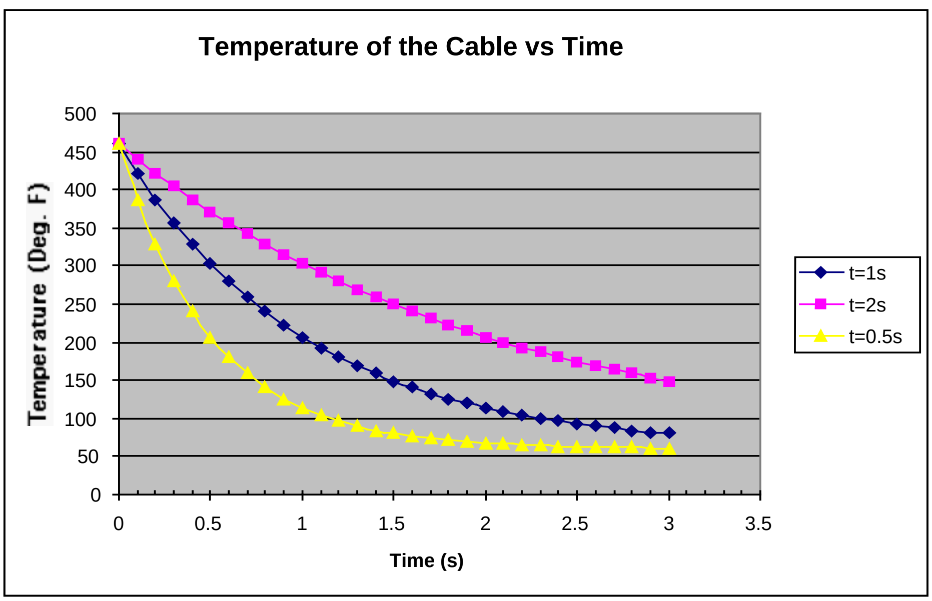
where $$\tau = \frac{Ac\rho_p}{\pi D h_{avg}}$$ Eq. (9)

Using Eqs. 8 & 9, we can predict the cooling characteristics of the polyethylene jacket.

If we rearrange Eq. (8) as

$$T_{cf} = e^{-t/\tau}(T_{ci} - T_\infty) + T_\infty$$ Eq. (10)

For the following temperature conditions T_{ci}=460°F, T_∞=60°F, we can plot the T_{cf} as a function of time. Plot shows cooling curves for τ=0.5s, τ=1s, τ=2s. From the plot we can determine that the cable will cool to 100°F in 2.3s if the τ=1s. If the time constant is large the cable will cool slowly, and if the time constant is small the cable will cool faster.



Challenge
To accurately predict the characteristics of the cooling curve, we need to determine an accurate time constant (Eq. 9). Time constant is dependent on the cross-sectional area of the polyethylene jacket, the properties of the polyethylene (density, specific heat capacity), the diameter of the cable, and the average heat transfer coefficient of the coolant.

Average Heat Transfer Coefficient
We can predict the heat transfer coefficient based on the coolant fluid properties by using the equations

$$h_{avg} = \frac{0.037 \mathrm{Re}_L^{4/5} \mathrm{Pr}^{1/3} k}{L} \qquad \text{Eq. (11)}$$

where

Re - Reynolds Number
Pr - Prandtl Number
k - thermal conductivity (W/mK)
L - length of cable

Attached spreadsheet outlines how to predict average heat transfer coefficient for a given coolant.

Properties of Polyethylene at 73.4^{o}F

	specific gravity	density (Kg/m^3)	Specific Heat (BTU/lb^{o}F)	Specific Heat (J/Kg.K)
Polyethylene				
Type I low density	0.925	925	0.55	2303
Type II medium density	0.94	940	0.55	2303
Type III high density	0.96	960	0.55	2303